

Base-4 Addition for Young People

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Numbers never stop. No matter how big a number you have, there always is a number that is just one more. To express numbers we will use symbols. We could make symbols look like anything, but it's not possible to keep track of a symbol for every number. So we'll create just a few symbols to start counting. I'll pick ♣, ◇, ♥, ♠. You can make others.

Next we'll need to put the symbols in order. Here's the order I choose.

♣	◇	♥	♠
first	second	third	fourth
0	1	2	3

With these four symbols we can count to three. It will be important to have a number that means no objects or zero. Now, how could we write a new number without making up a new symbol? One way is to use a ◇, a one, to represent one cycle through of all the symbols. We could write this ◇♣, one cycle and no more. Since there are only four symbols that we cycle through this means four. Now to show one more we could write ◇◇ to mean four and one more or five. Then ◇♥ for four and two more or six. This continues ◇♠ for seven, then ♥♣ for two four's or eight.

If we continue with this pattern then we can count as follows:

♣	◇	♥	♠
◇♣	◇◇	◇♥	◇♠
♥♣	♥◇	♥♥	♥♠
♠♣	♠◇	♠♥	♠♠
◇♣♣	◇♣◇	◇♣♥	◇♣♠
◇◇♣	◇◇◇	◇◇♥	◇◇♠

Once we cycle through the second place, or digit, we can then add another place or digit to remember that we cycled through four, four times. Thus the number ◇♣♣ stands for four four's or $4 + 4 + 4 + 4$ or 16. We repeat the pattern so that the next number is ◇♣◇, then ◇♣♥, ◇♣♠, ◇◇♣. The number ◇◇♣ is $16 + 4 = 20$. ◇◇♠ is $16 + 4 + 3$ which is 23.

Once we understand the numbers we can add(+) them. To add numbers, put them together and count how many you have. For example: ◇ + ◇ = ♥, ♥ + ◇ = ♠, ♠ + ◇ = ◇♣, ◇♣ + ♠ = ◇♠.

For small numbers, we can make a table that explains addition. This is called an addition table.

+	♣	◇	♡	♠	◇♣	◇◇
♣	♣	◇	♡	♠	◇♣	◇◇
◇	◇	♡	♠	◇♣	◇◇	◇♡
♡	♡	♠	◇♣	◇◇	◇♡	◇♠
♠	♠	◇♣	◇◇	◇♡	◇♠	♡♣
◇♣	◇♣	◇◇	◇♡	◇♠	♡♣	♡◇
◇◇	◇◇	◇♡	◇♠	♡♣	♡◇	♡♡

If we line our numbers up vertically, we can add by adding one digit at a time. If we end up with anything over ♠ we can carry a ◇ and add one to the next digit. Here are some examples:

$$\begin{array}{r}
 \diamond \clubsuit \\
 + \diamond \diamond \\
 \hline
 \heartsuit \diamond
 \end{array}
 \quad
 \begin{array}{r}
 \diamond \heartsuit \\
 \diamond \heartsuit \\
 + \diamond \heartsuit \\
 \hline
 \spadesuit \clubsuit
 \end{array}
 \quad
 \begin{array}{r}
 \diamond \spadesuit \\
 \heartsuit \spadesuit \\
 + \heartsuit \spadesuit \\
 \hline
 \diamond \diamond \heartsuit
 \end{array}
 \quad
 \begin{array}{r}
 \diamond \diamond \spadesuit \\
 \diamond \diamond \spadesuit \\
 + \diamond \diamond \spadesuit \\
 \hline
 \heartsuit \spadesuit \heartsuit
 \end{array}
 \quad
 \begin{array}{r}
 \diamond \diamond \heartsuit \\
 \diamond \diamond \heartsuit \\
 + \diamond \heartsuit \spadesuit \\
 \hline
 \spadesuit \diamond \diamond
 \end{array}$$

$$\begin{array}{r}
 \diamond \diamond \spadesuit \\
 \diamond \heartsuit \spadesuit \\
 + \diamond \diamond \diamond \\
 \hline
 \spadesuit \clubsuit \clubsuit
 \end{array}
 \quad
 \begin{array}{r}
 \heartsuit \diamond \spadesuit \\
 \heartsuit \diamond \spadesuit \\
 + \heartsuit \clubsuit \heartsuit \\
 \hline
 \diamond \clubsuit \heartsuit \diamond
 \end{array}
 \quad
 \begin{array}{r}
 \heartsuit \clubsuit \diamond \\
 \heartsuit \clubsuit \diamond \\
 + \spadesuit \heartsuit \diamond \\
 \hline
 \diamond \diamond \heartsuit \heartsuit
 \end{array}
 \quad
 \begin{array}{r}
 \diamond \diamond \spadesuit \clubsuit \\
 \heartsuit \spadesuit \clubsuit \\
 + \diamond \spadesuit \heartsuit \heartsuit \\
 \hline
 \heartsuit \heartsuit \diamond \diamond
 \end{array}
 \quad
 \begin{array}{r}
 \diamond \diamond \heartsuit \heartsuit \\
 \diamond \diamond \heartsuit \heartsuit \\
 + \heartsuit \heartsuit \heartsuit \heartsuit \\
 \hline
 \diamond \diamond \diamond \diamond \clubsuit
 \end{array}$$

Here are some for you to try.

$$\begin{array}{r} \diamond \heartsuit \\ + \diamond \diamond \\ \hline \end{array}$$

$$\begin{array}{r} \heartsuit \clubsuit \\ + \diamond \heartsuit \\ \hline \end{array}$$

$$\begin{array}{r} \heartsuit \diamond \\ + \spadesuit \heartsuit \\ \hline \end{array}$$

$$\begin{array}{r} \heartsuit \spadesuit \diamond \\ + \diamond \heartsuit \spadesuit \\ \hline \end{array}$$

$$\begin{array}{r} \diamond \spadesuit \spadesuit \\ + \heartsuit \clubsuit \diamond \\ \hline \end{array}$$

$$\begin{array}{r} \heartsuit \clubsuit \spadesuit \\ + \heartsuit \heartsuit \heartsuit \\ \hline \end{array}$$

$$\begin{array}{r} \diamond \diamond \heartsuit \\ + \heartsuit \heartsuit \diamond \\ \hline \end{array}$$

$$\begin{array}{r} \spadesuit \heartsuit \diamond \\ + \diamond \diamond \clubsuit \spadesuit \\ \hline \end{array}$$

$$\begin{array}{r} \diamond \heartsuit \spadesuit \clubsuit \\ + \heartsuit \clubsuit \heartsuit \spadesuit \\ \hline \end{array}$$

$$\begin{array}{r} \heartsuit \diamond \clubsuit \spadesuit \\ + \spadesuit \heartsuit \diamond \heartsuit \\ \hline \end{array}$$

$$\begin{array}{r} \heartsuit \heartsuit \clubsuit \clubsuit \\ + \heartsuit \heartsuit \heartsuit \heartsuit \\ \hline \end{array}$$

$$\begin{array}{r} \diamond \diamond \diamond \diamond \\ + \spadesuit \heartsuit \diamond \clubsuit \\ \hline \end{array}$$

$$\begin{array}{r} \diamond \clubsuit \clubsuit \diamond \\ + \diamond \heartsuit \clubsuit \spadesuit \\ \hline \end{array}$$

$$\begin{array}{r} \heartsuit \heartsuit \spadesuit \spadesuit \\ + \diamond \clubsuit \heartsuit \spadesuit \\ \hline \end{array}$$

$$\begin{array}{r} \spadesuit \spadesuit \spadesuit \spadesuit \\ + \diamond \diamond \heartsuit \spadesuit \\ \hline \end{array}$$